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VIBRATIONAL RESPONSE OF A BEAM WITH A BREATHING CRACK

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1. INTRODUCTION

Fatigue cracks often exist in structural members that are subjected to repeated loading, which will certainly lower the structural integrity. Many studies have been carried out on the dynamic response of fatigue cracks, in an attempt to find viable vibration methods for non-destructive inspection and health monitoring. The crack models used in these analyses fall largely into two categories: (1) open crack models and (2) opening and closing or breathing crack models. Most researchers have used open crack models in their studies and have claimed that the change in natural frequency might be a parameter used to detect the presence of cracks [1–4]. However, the assumption that cracks are always open in vibration is not realistic because compressive loads may close the cracks.

Recently, increasing efforts have focused on vibration analysis using opening and closing models to simulate a fatigue crack, as in Crespo *et al.* [5] and Prime and Shevitz [6]. Their fatigue crack model considers the bilinear behavior of an elastic crack and ignores the crack surface interference during fatigue. In their model, the structure has only two characteristic stiffness values: a larger value corresponding to the state of crack closing and a smaller value for crack opening. This fatigue crack model, however, only represents an idealized situation in which the crack has two perfectly flat surfaces and can only exist in the fully open or fully closed states. In reality, partial crack closure often occurs due to (1) roughness interference, (2) wedging by corrosion or wear debris, and (3) elastic constraint on the wake of the plastic zone. Therefore, the stiffness of a structure containing a real fatigue crack may change continuously with time as the load oscillates. A more general approach, employing many terms of a Fourier series to simulate the continuous

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change of stiffness in crack breathing, has been proposed by Abraham and Brandon [7]. However, the computational effort is not trivial.

In this paper, a simple non-linear fatigue crack model is developed. For simplicity, the dynamic behavior of a cracked beam vibrating at its first mode is analyzed using this fatigue crack model. Analyses are carried out in both time and frequency domains, which aim to identify the distinguishing features of the dynamic response associated with the existence of a fatigue crack.

2. THE BREATHING MODEL FOR A NON-LINEAR FATIGUE CRACK

Studies [8–11] have shown that the load–displacement response of a fatigue crack can be represented by the curve shown in Figure 1, where P_1, P_2, and P_3 signify the points when the crack is fully open, partially open and fully closed respectively. If one derives the stiffness of the structure from

$$k = \frac{\mathrm{d}P}{\mathrm{d}u} = k(t) \tag{1}$$

such a type of load–displacement relation leads to a continuous function such as

$$k = k(t). \tag{2}$$

Here, time t is chosen as the independent variable because the state of crack opening depends on the level of load, which varies with time due to vibration. For mathematical argument, the continuous stiffness in equation (1) can be decomposed into many terms of a Fourier series [7]. Examining the dynamic response of a fatigue crack at its first mode in a single-degree-of-freedom system, the stiffness may be expressed as

$$k(t) = k_0 + k_{\Delta c}(1 + \cos \omega_1 t) = k_1 + k_2(t) \tag{3}$$

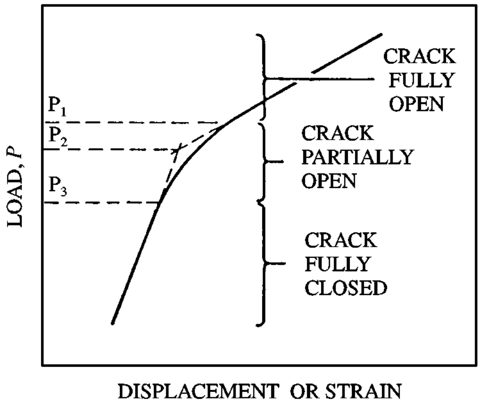

Figure 1. Schematic load–displacement curve [9].

where ω_1 is the crack breathing frequency, which is equal to the excitation frequency, $k_1 = k_0$ is the stiffness of the structure when the crack is fully open, and the amplitude of the stiffness change is given by

$$k_{\Delta c} = \tfrac{1}{2}(k_c - k_o), \tag{4}$$

where k_c is the stiffness when the crack is closed, and hence the stiffness change is

$$k_2(t) = k_{\Delta c}(1 + \cos \omega_1 t). \tag{5}$$

The above stiffness model assumes that the crack is completely closed when $\omega_1 t = 2l\pi$ ($l = 1, 2, 3, \ldots, l$, is any integer), then $k_2(2l\pi/\omega_1) = k_c - k_o$, $k = k_c$. When $\omega_1 t = (2l - 1)\pi (l = 1, 2, 3, \ldots, l$, is any integer number), then $k_2[(2l - 1)\pi/\omega_1] = 0$, $k = k_1$ and thus the crack is in the fully open state. Otherwise the crack is in a state of partial closure.

The present model simulates the change of the structural stiffness as a continuous function of time, i.e., when the crack opens and closes at a rate of ω_1. The coefficients k_0 and k_c are determined from the stiffness properties of the structure when the crack is completely open and completely closed respectively. When the crack is completely closed, the structure acts as one without a crack, and the stiffness k_c is determined using structural mechanics methods. When the crack is completely open, the stiffness k_0 can be determined using fracture mechanics. An example will be shown in section 3 for a cantilever beam.

Incorporating the breathing crack model into a single-degree-of-freedom system, the governing equation for forced vibration can be expressed as

$$m\ddot{u} + c\dot{u} + [k_o + k_{\Delta c}(1 + \cos \omega_1 t)]u = f, \tag{6}$$

where m is the mass, c is the damping coefficient, k is the stiffness, f is the exciting force and u is the displacement.

3. ANALYSIS OF A CRACKED BEAM

For the sake of simplicity, a cantilever beam is modelled as a one-degree-of-freedom lumped parameter system. This simplified model simulates the beam vibrating at its first mode. Both open crack and breathing crack models will be incorporated in the system to identify the difference in their responses.

3.1. MODELLING

The cantilever beam under analysis is shown in Figure 2. For this beam, equation (3) could be rewritten as

$$k(t) = EI[\alpha + \beta(1 + \cos \omega_1 t)] = k_1 + k_2(t), \tag{7}$$

where α and β are constants. $k_1 = k_0 = EI\alpha$ is the stiffness of the beam when it is fully open. And

$$k_2(t) = EI\beta(1 + \cos \omega_1 t). \tag{8}$$

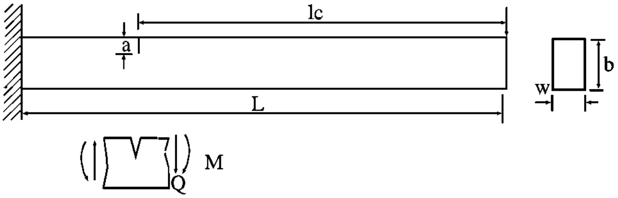

Figure 2. Cantilever beam with a crack.

The change in the flexibility of a cracked beam can be derived from the equation developed by Dimarogonas and Paipetis [12] as

$$c_{ii} = \frac{\partial u_i}{\partial P_i} = w \frac{\partial^2}{\partial P_i^2} \int_0^a J \, \mathrm{d}a^* = \frac{72 l_c^2 \pi (1 - \nu^2)}{E w b^4} \phi, \tag{9}$$

where ϕ is given by

$$\phi = 19.60 \frac{a^{10}}{b^8} - 40.69 \frac{a^9}{b^7} + 47.04 \frac{a^8}{b^6} - 32.99 \frac{a^7}{b^5} + 20.30 \frac{a^6}{b^4} - 9.98 \frac{a^5}{b^3} + 4.60 \frac{a^4}{b^2} - 1.05 \frac{a^3}{b} + 0.63 a^2, \tag{10}$$

where a is the depth of the crack and b is the depth of the beam as shown in Figure 2. Hence, the total flexibility of the beam containing an open crack is given by [13]

$$c_{open} = c_{ii} + c_{no\ crack}, \tag{11}$$

where $c_{no\ crack}$ is the beam's flexibility without a crack.

The stiffness of an open crack is $k_0 = 1/c_{open}$. The stiffness of a closed crack is $k_c = 1/c_{no\ crack}$. Using the above relations and formulas, the coefficients α and β can easily be determined:

$$\alpha = k_o/EI, \quad \beta = (k_c - k_o)/2EI. \tag{12}$$

The equation of motion for this model can then be expressed as

$$m\ddot{u} + c\dot{u} + EI[\alpha + \beta(1 + \cos \omega_1 t)]u = F \sin \omega_1 t. \tag{13}$$

3.2. RESULTS

For a numerical example of a rectangular cantilever beam containing a transverse crack, as shown in Figure 2, the dimensions are chosen as $L = 9$ m, $w = 0{\cdot}15$ m and $b = 0{\cdot}26$ m [14]. Young's modulus is assumed to be $E = 206 \times 10^9$ N/m^2 and l_c is taken as $0{\cdot}9L$. Then, the generalized mass and stiffness of the beam can be derived as [15]

$$k^* = k_c = EI\pi^4/32L^3, \quad m^* = 0{\cdot}228 m' L, \tag{14}$$

where m' is mass of unit length. The natural frequency of the beam without crack is 17·4 rad/s (2·77 Hz).

Using the above parameters, the constants α and β were calculated and the equation of motion was solved numerically using the fourth/fifth order Runge–Kutta method [16]. The Hanning window was introduced over the response signal and fast Fourier transform was used to compute a frequency response function (FRF).

Figure 3 shows the natural frequency ratio as a function of the crack severity for an open crack and a fatigue crack. It can be seen that the natural frequency for the beam with a fatigue crack is higher than that with an open crack and hence the frequency shift should be smaller in a fatigue-cracked beam. This implies that: (1) a fatigue crack is more difficult to detect by frequency monitoring, and (2) if one detects the presence of a crack by a frequency shift of the structure, the crack may have penetrated to greater depth under fatigue loading conditions than predicted by an open-crack model. Therefore, using an open-crack model for crack detection tends to give rise to dangerous conclusions.

Figure 4 shows the displacement response in forced vibration at a crack severity of $a/b = 0{\cdot}3$, with a damping ratio being equal to $\zeta = 0{\cdot}01$, where damping ratio is defined as $\zeta = c/(2m\omega)$. It may be noticed that after approximately 1 s of reaching the steady state, the displacement curve of the beam with a fatigue crack falls between those of the uncracked beam and the beam with an open crack.

In the frequency domain, the FRF shows some interesting features for (a) an open crack and (b) a fatigue crack, as shown in Figure 5. It can be seen that the resonance peak of the open crack is very smooth, but side peaks appear beside the resonance peak of the fatigue crack. These side peaks indicate the non-linear nature of the

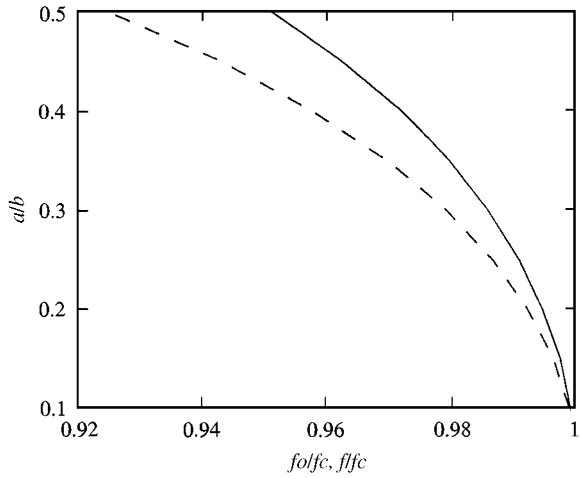

Figure 3. Frequency change with crack severity change for breathing crack (—,f/f_c) and open crack (- -,f_o/f_c). f_0 is the frequency of the beam with an open crack, f is the frequency of the beam with a breathing crack, f_c is the frequency of the beam.

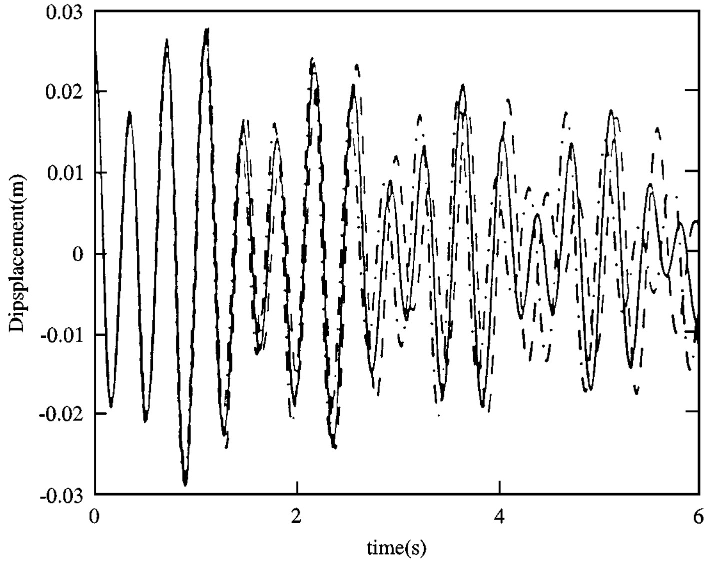

Figure 4. Damped forced vibration with $a/b = 0{\cdot}3$, $\zeta = 0{\cdot}01$ and crack breathing frequency 2 Hz; open crack (- -); breathing crack (—); no crack (- ·).

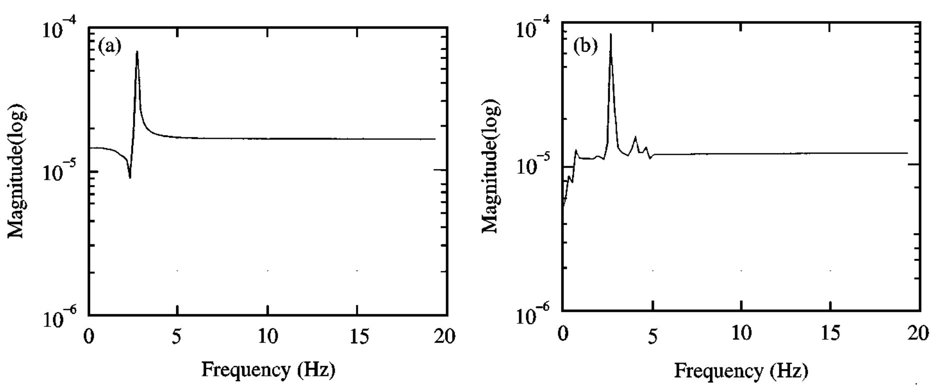

Figure 5. Frequency response function for damped forced vibration with $\zeta = 0{\cdot}01$, $n = 1$, $a/b = 0{\cdot}3$: (a) open crack, (b) opening and closing crack.

response of a fatigue crack. The results of this simulation indicate that a fatigue crack can cause the frequency response to be non-linear, which has been observed experimentally and in computer simulations [17]. The presence of side peaks may be used as a feature to recognize the presence of fatigue cracks.

Figure 6 shows the phase plane diagrams with the initial conditions of $u = 0{\cdot}025$ m and $\dot{u} = 0$ m/s for a damped system with $\zeta = 0{\cdot}01$. From Figure 6, it may be seen that the phase plane diagram plots for both open and fatigue crack are similar. No recognizable differences are observed from the phase plane plots for "open" and "fatigue" cracks as suggested in an earlier study [18].

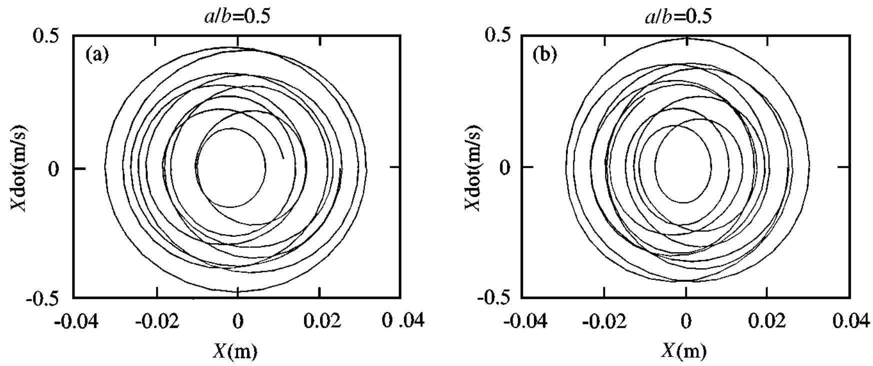

Figure. 6. Phase plane plots for damped vibration with $\zeta = 0{\cdot}01$: (a) open crack, (b) opening and closing crack.

4. CONCLUSION

Our analysis, using a continuous breathing crack model, has shown that the natural frequency reduction for a fatigue crack (breathing crack) is much smaller than for an open crack. This means (1) that fatigue cracks would be difficult to recognize by frequency monitoring and (2) that crack detection by an open crack model would underestimate the crack severity if the crack was actually growing under fatigue loading conditions. Another interesting result is that side peaks appear in the frequency response functions of a fatigue crack near the resonance peak.

The above observations suggest that detection of fatigue cracks should be more reliably based on non-linear features of FRF, rather than the natural frequency shift. Indeed, pronounced anti-resonance frequency shifts and super/sub-harmonic vibration phenomena have been observed in experimental study of naturally grown fatigue cracks [19].

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